

Introduction to game coloring

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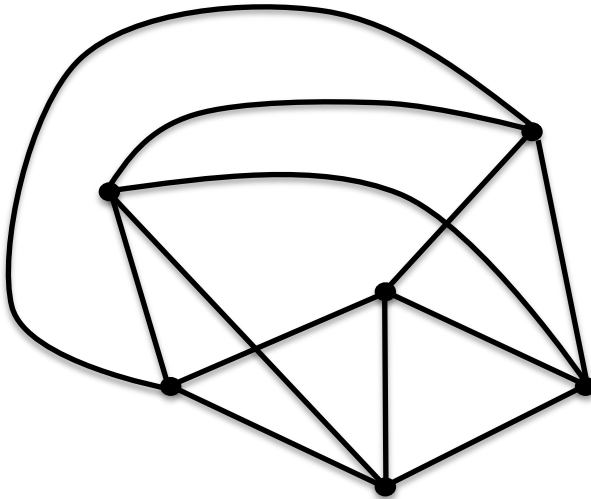
CNRS – Institut Fourier

Maths à Modeler – Université Grenoble Alpes

Game coloring ...

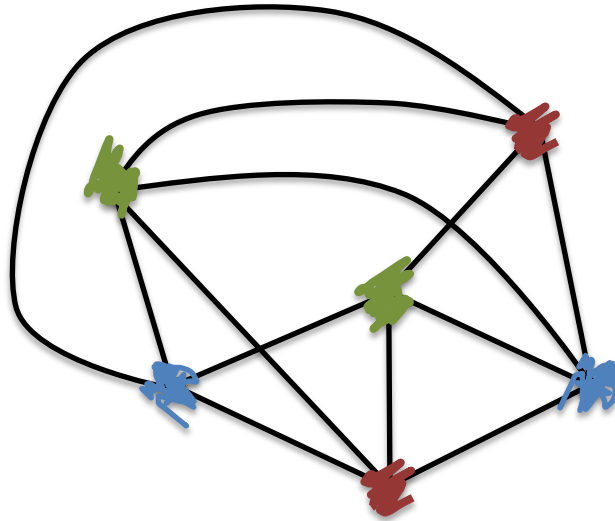


Choose a set of colors



Take your favorite graph

Graph coloring ...

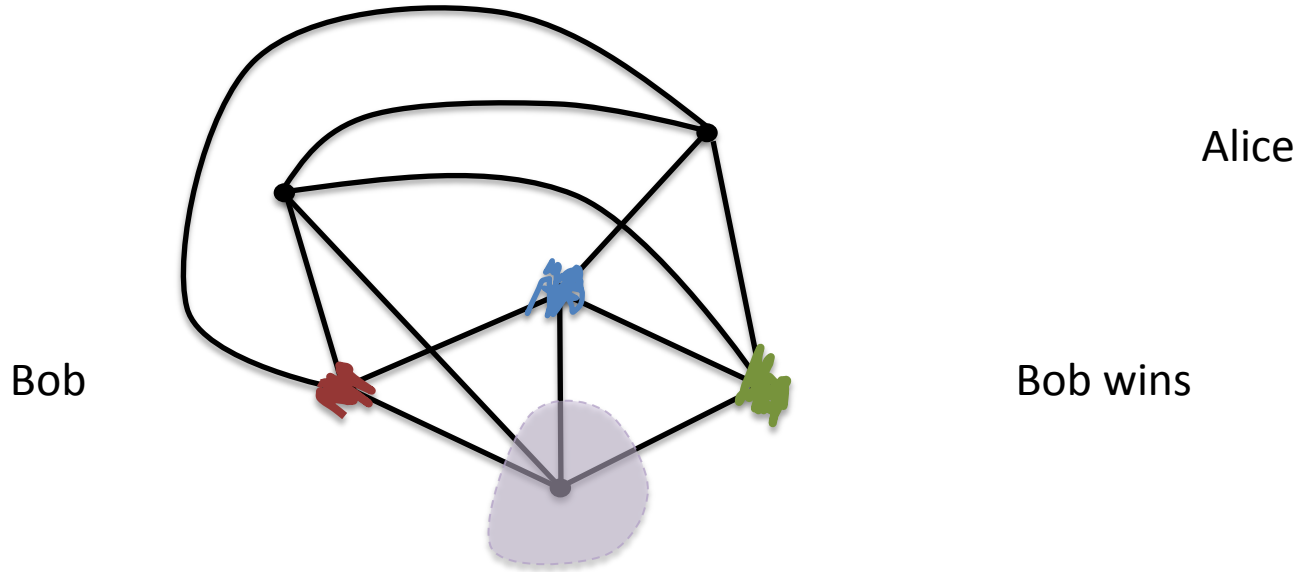


$\chi(G)$
Minimum number of colors needed
to color G .

Play Game coloring ...



Alice versus Bob

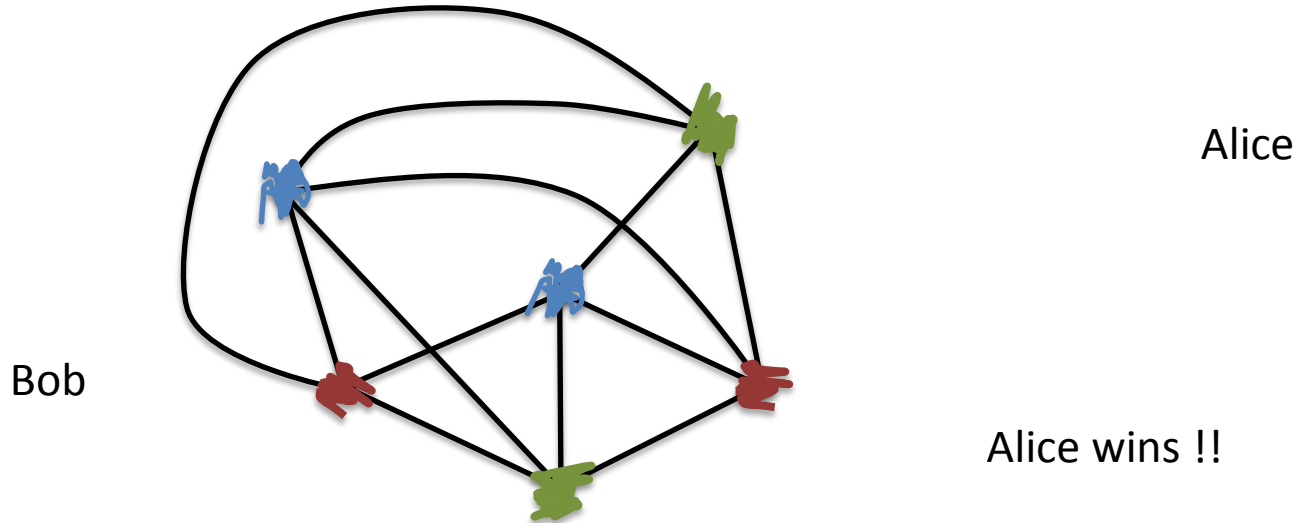


Brams via Gardner 1981

Play Game coloring ...



Alice versus Bob



Brams via Gardner 1981

Game coloring analysis...

Depend who start ...

So fixe that Alice starts

*Alice, the Gentel and
Bob, the Rascal*

With enough colors Alice wins

n

How many colors for insuring the Alice's victory ?

The game chromatic number $\chi_g(G)$

Game Coloring versus Coloring

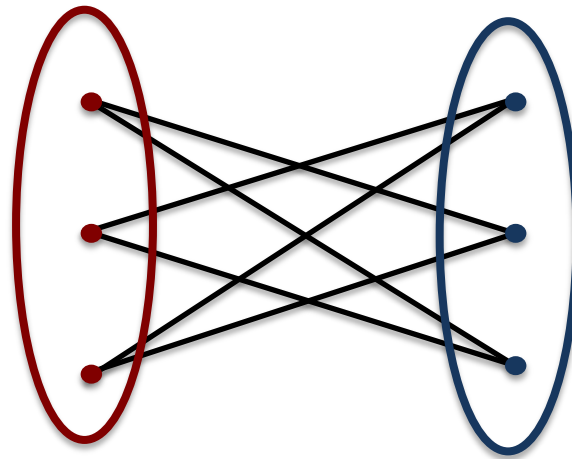
Clearly, $\chi(G) \leq \chi_g(G) !!$

Ok, but does there exist a constant C such that
 $\chi(G) \geq \chi_g(G) + C$?

$$\chi(G) \geq \chi_g(G) + C ?$$

Try with $\chi(G) = 2$

Hint ...



Game Coloring versus Coloring

Clearly, $\chi(G) \leq \chi_g(G) !!$

~~Ok, but does there exist a constant C such that
 $\chi(G) \geq \chi_g(G) + C$?~~

NO !!

Game Coloring on Trees

Start with path ...

$$X_g(P_n) = ??$$

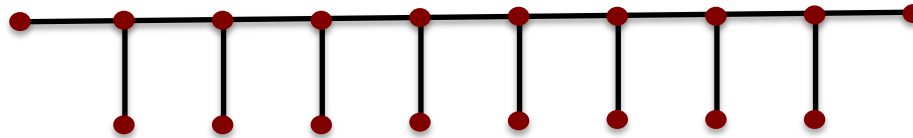
Can we deduce something for cycle ...

$$X_g(C_n) = ??$$

Game Coloring on Trees

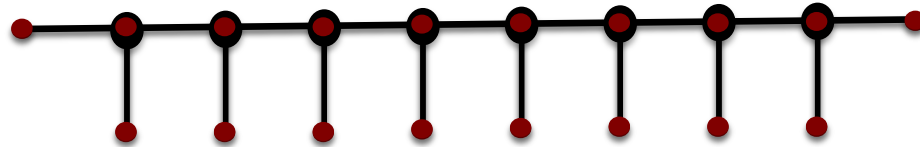
Paths too easy !!

OK try some *caterpillars* ...



Game Coloring on Caterpillars

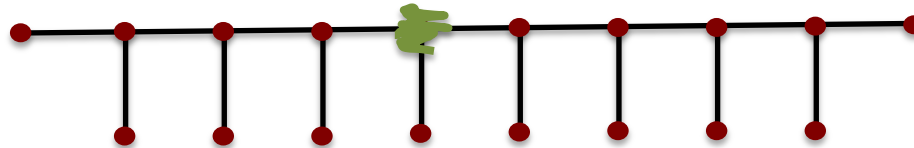
Take 3 colors



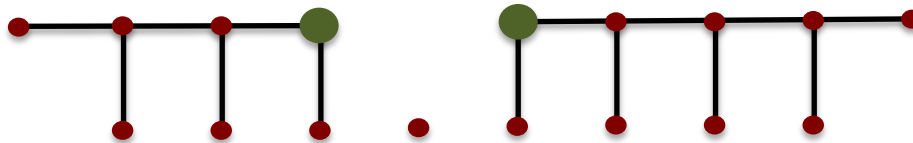
● means brittle vertex ...


Game Coloring on Caterpillars

Take 3 colors

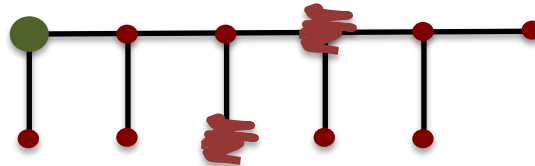


Alice plays



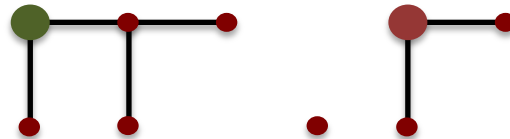
● Means color  forbidden on brittle vertex !!

Game Coloring on Caterpillars



Bob play

Alice answer

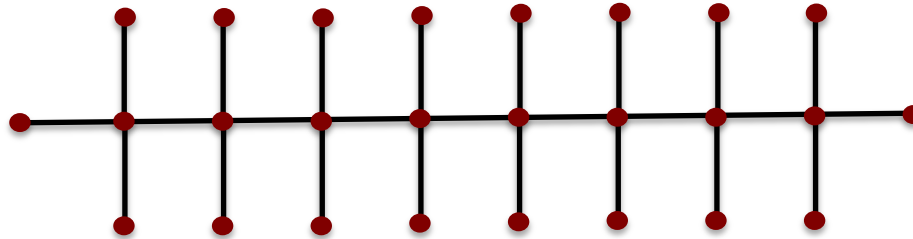


Conclude by induction ...

Game Coloring on Trees

Caterpillars too easy !!

Thanks to Simone et al.¹ there is Caterpillar C such that $X_g(C) > 3$



Game coloring known stuff...

Trees are 4 game-colorable...

Faigle, Kierstead, Kern and Trotter 1993

Constant bounded for planar graphs

e.g. Zhu 2008, for fixed genus

Monotone or not ???

PSPACE-complete (for 3)

Boedlander 1991

With enough colors Alice wins

$$\chi(G) + \frac{n}{2}$$

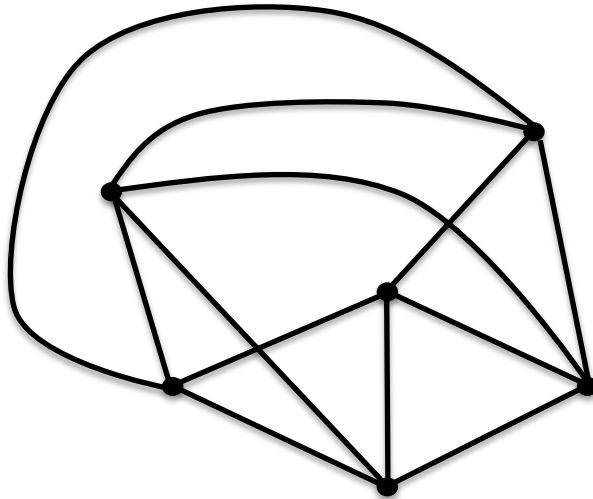
Nordhauss-Gaddum inequalities $\chi_g(G)$

Charpentier, Furtado and Gravier 2016

Game coloring known stuff...



Not hereditary ...



Bob wins when Alice starts i.e. $\chi_g(G) > 3$
Alice wins when Alice starts i.e. $\chi_g(G) \leq 3$