Maximum cut and Steiner tree restricted to interval graphs and related families

Celina Miraglia Herrera de Figueiredo



CANADAM 2021

Based on

Maximum cut on interval graphs of interval count four is NP-complete with Alexsander A. de Melo, Fabiano S. Oliveira, Ana Silva arxiv.org/abs/2012.09804

Revising Johnson's table for the 21st century with Alexsander A. de Melo, Diana Sasaki, Ana Silva arxiv.org/abs/2104.14395





The complexity of hard graph problems forty years later

Celina Miraglia Herrera de Figueiredo



Workshop ParGO 20+50, Universidade Federal do Ceará, agosto 2019

Knuth – Garey – Johnson



The Guide – Computers and Intractability



"Despite that 23 years have passed since its publication, I consider Garey and Johnson the single most important book on my office bookshelf. Every computer scientist should have this book on their shelves as well. NP-completeness is the single most important concept to come out of theoretical computer science and no book covers it as well as Garey and Johnson."

Lance Fortnow, "Great Books: Computers and Intractability: A Guide to the Theory of NP-Completeness"

Ongoing Guide – Graph Restrictions and Their Effect

GRAPH CLASS	ME	MBER	INI	SET	CLIC	QUE	CLI	PAR	Сн	NUM	CHR	IND	HAN	ICIR	Do	4SET	MAX	KCUT	STT	REE	GRA	Iso
Trees/Forests	Р	[T]	Р	[GJ]	Р	[T]	Р	[GJ]	Р	[T]	Р	[GJ]	Р	[T]	Р	[GJ]	Р	[GJ]	Р	[T]	Р	[GJ]
Almost Trees (k)	Р		P	[24]	Р	[T]	P?		P?		P ?		P ?		Р	[45]	P ?		P?		P ?	
Partial k-Trees	Р	[2]	P	[1]	Р	[T]	P?		Р	[1]	O ?		Р	[3]	Р	[3]	P ?		P?		O ?	
Bandwidth-k	Р	[68]	P	[64]	Р	[T]	P?		Р	[64]	P ?		P ?		Р	[64]	Р	[64]	P?		Р	[58]
Degree-k	Р	[T]	Ν	[GJ]	Р	[T]	Ν	[GJ]	Ν	[GJ]	Ν	[49]	Ν	[GJ]	Ν	[GJ]	Ν	[GJ]	Ν	[GJ]	Р	[58]
Planar	Р	[GJ]	Ν	[GJ]	Р	[T]	Ν	[10]	Ν	[GJ]	0		Ν	[GJ]	Ν	[GJ]	Р	[GJ]	Ν	[35]	Р	[GJ]
Series Parallel	Р	[79]	P	[75]	Р	[T]	P?		Р	[74]	Р	[74]	Р	[74]	Р	[54]	Р	[GJ]	Р	[82]	Р	[GJ]
Outerplanar	Р		P	[6]	Р	[T]	Р	[6]	Р	[67]	Р	[67]	Р	[T]	Р	[6]	Р	[GJ]	Р	[81]	Р	[GJ]
Halin	Р		P	[6]	Р	[T]	Р	[6]	Р	[74]	Р	[74]	Р	[T]	Р	[6]	Р	[GJ]	P ?		Р	[GJ]
k-Outerplanar	Р		P	[6]	Р	[T]	Р	[6]	Р	[6]	O ?		Р	[6]	Р	[6]	Р	[GJ]	P ?		Р	[GJ]
Grid	Р		P	[GJ]	Р	[T]	Р	[GJ]	Р	[T]	Р	[GJ]	Ν	[51]	Ν	[55]	Р	[T]	Ν	[35]	Р	[GJ]
K _{3.3} -Free	Р	[4]	N	[GJ]	Р	[T]	Ν	[10]	Ν	[GJ]	O ?		Ν	[GJ]	Ν	[GJ]	Р	[5]	Ν	[GJ]	O ?	
Thickness-k	Ν	[60]	N	[GJ]	Р	[T]	Ν	[10]	Ν	[GJ]	Ν	[49]	Ν	[GJ]	Ν	[GJ]	Ν	[7]	Ν	[GJ]	O ?	
Genus-k	Р	[34]	Ν	[GJ]	Р	[T]	Ν	[10]	Ν	[GJ]	O ?		Ν	[GJ]	Ν	[GJ]	O ?		Ν	[GJ]	Р	[61]
Perfect	0!		P	[42]	Р	[42]	Р	[42]	Р	[42]	0?		Ν	[1]	Ν	[14]	O ?		Ν	[GJ]	I	[GJ]
Chordal	Р	[76]	P	[40]	Р	[40]	Р	[40]	Р	[40]	O ?		Ν	[22]	Ν	[14]	O ?		Ν	[83]	I	[GJ]
Split	Р	[40]	P	[40]	Р	[40]	Р	[40]	Р	[40]	O ?		Ν	[22]	Ν	[19]	O ?		Ν	[83]	I	[15]
Strongly Chordal	Р	[31]	P	[40]	Р	[40]	Р	[40]	Р	[40]	O ?		O ?		Р	[32]	O ?		Р	[83]	O ?	
Comparability	Р	[40]	P	[40]	Р	[40]	Р	[40]	Р	[40]	O ?		Ν	[1]	Ν	[28]	O ?		Ν	[GJ]	I	[GJ]
Bipartite	Р	[T]	P	[GJ]	Р	[T]	Р	[GJ]	Р	[T]	Ρ	[GJ]	Ν	[1]	Ν	[28]	Ρ	[T]	Ν	[GJ]	I	[GJ]
Permutation	Р	[40]	P	[40]	Р	[40]	Р	[40]	Р	[40]	O ?		0		Р	[33]	O ?		Р	[23]	Р	[21]
Cographs	Р	[T]	Р	[40]	Р	[40]	Р	[40]	Р	[40]	O ?		Р	[25]	Р	[33]	O ?		Р	[23]	Р	[25]
Undirected Path	Р	[39]	Р	[40]	Р	[40]	Р	[40]	Р	[40]	0?		O ?		Ν	[16]	0?		O ?		I	[GJ]
Directed Path	Р	[38]	P	[40]	Р	[40]	Р	[40]	Р	[40]	O ?		O ?		Р	[16]	O ?		Р	[83]	O ?	
Interval	Р	[17]	P	[44]	Р	[44]	Р	[44]	Р	[44]	O ?		Р	[53]	Р	[16]	O ?		Р	[83]	Р	[57]
Circular Arc	Р	[78]	P	[44]	Р	[50]	Р	[44]	Ν	[36]	O ?		O ?		Р	[13]	O ?		Р	[83]	O ?	
Circle	Р	[71]	P	[GJ]	Р	[50]	O ?		Ν	[36]	O ?		Р	[12]	O ?		O ?		Р	[70]	O ?	
Proper Circ. Arc	Р	[77]	P	[44]	Р	[50]	Р	[44]	Р	[66]	O ?		Р	[12]	Р	[13]	O ?		Р	[83]	O ?	
Edge (or Line)	Р	[47]	Р	[GJ]	Р	[T]	Ν	[GJ]	Ν	[49]	O ?		Ν	[11]	Ν	[GJ]	O ?		Ν	[70]	I	[15]
Claw-Free	Р	[T]	Р	[63]	O ?		Ν	[GJ]	Ν	[49]	O ?		Ν	[11]	Ν	[GJ]	O ?		Ν	[70]	I	[15]

Containment relations for classes



The updated NP-Completeness Column: An Ongoing Guide table 35 years later

GRAPH CLASS	М	EMBER	In	dSet	CI	.IQUE	CI	.IPAR	C	hrNum	Сн	rInd	H	AMCIR	D	OMSET	MA	хCut	Sт	TREE	G	RAPHISO
TREES/FORESTS	Ρ	[T]	Ρ	[GJ]	Ρ	[T]	Ρ	[GJ]	Ρ	[T]	Р	[GJ]	Ρ	[T]	Ρ	[GJ]	Р	[GJ]	Ρ	[T]	Ρ	[GJ]
Almost Trees (k)	Р	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[16]	Ρ	[5]	Ρ	[19]	Р	[5]	Ρ	[5]	Ρ	[20]	Ρ	[76]	Ρ	[19]
PARTIAL k-TREES	Р	[OG]	Р	[5]	Р	[T]	Ρ	[16]	Ρ	[5]	Ρ	[19]	Р	[5]	Ρ	[5]	Ρ	[20]	Ρ	[76]	Р	[19]
BANDWIDTH-k	Р	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[16]	Ρ	[5]	Ρ	[19]	Р	[5]	Ρ	[5]	Ρ	[OG]	Р	[76]	Ρ	[OG]
Degree-k	Ρ	[T]	Ν	[GJ]	Ρ	[T]	Ν	[29]	Ν	[GJ]	Ν	[OG]	Ν	[GJ]	Ν	[GJ]	Ν	[GJ]	Ν	[GJ]	Ρ	[OG]
PLANAR	Р	[GJ]	Ν	[GJ]	Ρ	[T]	N	[77]	N	[GJ]	0		N	[GJ]	N	[GJ]	Р	[GJ]	Ν	[OG]	Р	[GJ]
SERIES PARALLEL	Р	[OG]	Ρ	[OG]	Ρ	[T]	Р	[16]	Ρ	[5]	Ρ	[19]	Р	[5]	Ρ	[OG]	Ρ	[GJ]	Ρ	[OG]	Ρ	[GJ]
OUTERPLANAR	Р	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Р	[T]	Ρ	[OG]	Ρ	[GJ]	Ρ	[OG]	Ρ	[GJ]
HALIN	Р	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[OG]	Ρ	[5]	Ρ	[19]	Р	[T]	Ρ	[OG]	Ρ	[GJ]	Ρ	[118]	Ρ	[GJ]
k-Outerplanar	Р	[OG]	Ρ	[OG]	Ρ	[T]	Р	[OG]	Ρ	[5]	Ρ	[19]	Р	[OG]	Ρ	[OG]	Ρ	[GJ]	Ρ	[76]	Ρ	[GJ]
Grid	Р	[OG]	Ρ	[GJ]	Ρ	[T]	Р	[GJ]	Ρ	[T]	Р	[GJ]	Ν	[OG]	Ν	[32]	Ρ	[T]	Ν	[OG]	Ρ	[GJ]
$K_{3,3}$ -Free [*]	Р	[OG]	Ν	[GJ]	Ρ	[T]	Ν	[77]	Ν	[GJ]	0?		Ν	[GJ]	Ν	[GJ]	Ρ	[OG]	Ν	[GJ]	Ρ	[40]
THICKNESS-k	N	[OG]	N	[GJ]	Р	[T]	Ν	[77]	Ν	[GJ]	Ν	[OG]	Ν	[GJ]	Ν	[GJ]	Ν	[119]	Ν	[GJ]	τ.	Prop. 3
GENUS-k	Ρ	[OG]	N	[GJ]	Ρ	[T]	Ν	[77]	Ν	[GJ]	0?		Ν	[GJ]	N	[GJ]	0?		Ν	[GJ]	Ρ	[OG]
Perfect	Р	[34]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	N	[28]	N	[OG]	Ν	[OG]	N	[20]	Ν	[GJ]	I.	[83]
CHORDAL	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ν	[92]	Ν	[OG]	Ν	[20]	Ν	[OG]	1	[83]
Split	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Р	[OG]	Ρ	[OG]	0?		Ν	[92]	Ν	[OG]	Ν	[20]	Ν	[OG]	1	[107]
STRONGLY CHORDAL	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ν	[92]	Ρ	[OG]	Ν	[108]	Ρ	[OG]	Γ.	[111]
COMPARABILITY	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ν	[28]	Ν	[OG]	Ν	[93]	Ν	[101]	Ν	[GJ]	1	[23]
BIPARTITE	Р	[T]	Ρ	[GJ]	Ρ	[T]	Р	[GJ]	Ρ	[T]	Ρ	[GJ]	Ν	[OG]	Ν	[93]	Ρ	[T]	Ν	[GJ]	Т.	[23]
PERMUTATION	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Р	[44]	Ρ	[OG]	0?		Ρ	[OG]	Ρ	[OG]
Cographs	Ρ	[T]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ρ	[OG]	Ρ	[OG]	Р	[20]	Ρ	[OG]	Ρ	[OG]
UNDIRECTED Path	Ρ	[OG]	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ν	[13]	N	[OG]	Ν	[20]	N	Thm. 4	Ľ.	[23]
DIRECTED PATH	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Р	[OG]	Ρ	[OG]	0?		Ν	[98]	Ρ	[OG]	Ν	[1]	Ρ	[OG]	Ρ	[7]
INTERVAL	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Р	[OG]	Ρ	[OG]	Ν	[1]	Ρ	[OG]	Ρ	[OG]
CIRCULAR ARC	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Р	[OG]	Ν	[OG]	0?		Ρ	[105]	Ρ	[OG]	Ν	[1]	Ρ	[11]	Р	[79]
CIRCLE	Р	[OG]	Ρ	[GJ]	Ρ	[OG]	Ν	[73]	Ν	[OG]	0?		Ν	[39]	Ν	[71]	Ν	[26]	Ρ	[<u>OG</u>]	Ρ	[68]
PROPER CIRC. ARC	Р	[OG]	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ρ	[OG]	Ρ	[OG]	0?		Ρ	[11]	Ρ	[81]
Edge (or Line)	Р	[OG]	Р	[GJ]	Ρ	[T]	Ν	[94]	N	[OG]	Ν	[28]	N	[OG]	Ν	[GJ]	Ρ	[59]	Ν	[18]	I.	[OG]
CLAW-FREE	Ρ	[T]	Р	[OG]	Ν	[102]	Ν	[84]	Ν	[OG]	Ν	[28]	Ν	[OG]	Ν	[GJ]	Ν	[20]	Ν	[18]	L.	[OG]

Maximum cut and Steiner tree

We focus on columns MaxCut and StTree

Full dichotomy for StTree column: the problem is NP-complete when restricted to UNDIRECTED PATH graphs

Complexity of maximum cut on interval graphs R. Adhikary, K. Bose, S. Mukherjee, and B. Roy. arxiv.org/abs/2006.00061

MaxCut restricted to interval graphs with bounded interval count is NP-complete

The grained gadget of Adhikary et al.



Our escalator gadget



The updated NP-Completeness Column: An Ongoing Guide table 35 years later

GRAPH CLASS	М	EMBER	In	dSet	CI	.IQUE	CI	.IPAR	C	hrNum	Сн	rInd	H	AMCIR	D	OMSET	MA	хCut	Sт	TREE	G	RAPHISO
TREES/FORESTS	Ρ	[T]	Ρ	[GJ]	Ρ	[T]	Ρ	[GJ]	Ρ	[T]	Р	[GJ]	Ρ	[T]	Ρ	[GJ]	Р	[GJ]	Ρ	[T]	Ρ	[GJ]
Almost Trees (k)	Р	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[16]	Ρ	[5]	Ρ	[19]	Р	[5]	Ρ	[5]	Ρ	[20]	Ρ	[76]	Ρ	[19]
PARTIAL k-TREES	Р	[OG]	Р	[5]	Ρ	[T]	Р	[16]	Ρ	[5]	Ρ	[19]	Р	[5]	Ρ	[5]	Ρ	[20]	Ρ	[76]	Р	[19]
BANDWIDTH-k	Р	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[16]	Ρ	[5]	Ρ	[19]	Р	[5]	Ρ	[5]	Ρ	[OG]	Р	[76]	Ρ	[OG]
Degree-k	Ρ	[T]	Ν	[GJ]	Ρ	[T]	Ν	[29]	Ν	[GJ]	Ν	[OG]	Ν	[GJ]	Ν	[GJ]	Ν	[GJ]	Ν	[GJ]	Ρ	[OG]
PLANAR	Р	[GJ]	Ν	[GJ]	Ρ	[T]	N	[77]	N	[GJ]	0		N	[GJ]	N	[GJ]	Р	[GJ]	Ν	[OG]	Р	[GJ]
SERIES PARALLEL	Р	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[16]	Ρ	[5]	Ρ	[19]	Р	[5]	Ρ	[OG]	Ρ	[GJ]	Ρ	[OG]	Ρ	[GJ]
OUTERPLANAR	Ρ	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Р	[T]	Ρ	[OG]	Ρ	[GJ]	Ρ	[OG]	Ρ	[GJ]
HALIN	Ρ	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[OG]	Ρ	[5]	Ρ	[19]	Р	[T]	Ρ	[OG]	Ρ	[GJ]	Ρ	[118]	Ρ	[GJ]
k-Outerplanar	Р	[OG]	Ρ	[OG]	Ρ	[T]	Ρ	[OG]	Ρ	[5]	Ρ	[19]	Р	[OG]	Ρ	[OG]	Ρ	[GJ]	Ρ	[76]	Ρ	[GJ]
Grid	Р	[OG]	Ρ	[GJ]	Ρ	[T]	Ρ	[GJ]	Ρ	[T]	Р	[GJ]	Ν	[OG]	Ν	[32]	Ρ	[T]	Ν	[OG]	Ρ	[GJ]
$K_{3,3}$ -Free [*]	Р	[OG]	Ν	[GJ]	Ρ	[T]	Ν	[77]	Ν	[GJ]	0?		Ν	[GJ]	Ν	[GJ]	Ρ	[OG]	Ν	[GJ]	Ρ	[40]
THICKNESS-k	N	[OG]	N	[GJ]	Ρ	[T]	Ν	[77]	Ν	[GJ]	Ν	[OG]	Ν	[GJ]	Ν	[GJ]	Ν	[119]	Ν	[GJ]	τ.	Prop. 3
GENUS-k	Ρ	[OG]	N	[GJ]	Ρ	[T]	Ν	[77]	Ν	[GJ]	0?		Ν	[GJ]	N	[GJ]	0?		Ν	[GJ]	Ρ	[OG]
Perfect	Р	[34]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	N	[28]	Ν	[OG]	Ν	[OG]	N	[20]	Ν	[GJ]	I.	[83]
CHORDAL	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ν	[92]	Ν	[OG]	Ν	[20]	Ν	[OG]	1	[83]
Split	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Р	[OG]	Ρ	[OG]	0?		Ν	[92]	Ν	[OG]	Ν	[20]	Ν	[OG]	1	[107]
STRONGLY CHORDAL	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ν	[92]	Ρ	[OG]	Ν	[108]	Ρ	[OG]	Γ.	[111]
COMPARABILITY	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ν	[28]	Ν	[OG]	Ν	[93]	Ν	[101]	Ν	[GJ]	1	[23]
BIPARTITE	Р	[T]	Ρ	[GJ]	Ρ	[T]	Р	[GJ]	Ρ	[T]	Ρ	[GJ]	Ν	[OG]	Ν	[93]	Ρ	[T]	Ν	[GJ]	Т.	[23]
PERMUTATION	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Р	[44]	Ρ	[OG]	0?		Ρ	[OG]	Ρ	[OG]
Cographs	Ρ	[T]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ρ	[OG]	Ρ	[OG]	Р	[20]	Ρ	[OG]	Ρ	[OG]
UNDIRECTED Path	Ρ	[OG]	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Ν	[13]	N	[OG]	Ν	[20]	N	Thm. 4	Ľ.	[23]
DIRECTED PATH	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Р	[OG]	Ρ	[OG]	0?		Ν	[98]	Ρ	[OG]	Ν	[1]	Ρ	[OG]	Ρ	[7]
INTERVAL	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Р	[OG]	Ρ	[OG]	Ν	[1]	Ρ	[OG]	Ρ	[OG]
CIRCULAR ARC	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Р	[OG]	Ν	[OG]	0?		Р	[105]	Ρ	[OG]	Ν	[1]	Ρ	[11]	Р	[79]
CIRCLE	Р	[OG]	Ρ	[GJ]	Ρ	[OG]	Ν	[73]	Ν	[OG]	0?		Ν	[39]	Ν	[71]	Ν	[26]	Ρ	[<u>OG</u>]	Ρ	[68]
PROPER CIRC. ARC	Р	[OG]	Р	[OG]	Ρ	[OG]	Ρ	[OG]	Ρ	[OG]	0?		Р	[OG]	Ρ	[OG]	0?		Ρ	[11]	Ρ	[81]
Edge (or Line)	Р	[OG]	Р	[GJ]	Ρ	[T]	Ν	[94]	N	[OG]	Ν	[28]	N	[OG]	Ν	[GJ]	Р	[59]	Ν	[18]	I.	[OG]
CLAW-FREE	Ρ	[T]	Р	[OG]	Ν	[102]	Ν	[84]	Ν	[OG]	Ν	[28]	Ν	[OG]	Ν	[GJ]	Ν	[20]	Ν	[18]	L.	[OG]

The chromatic index of split-interval graphs

Luis Gustavo Gonzaga, Sheila Almeida, Cândida Silva and Jadder Cruz

presented last week at LAGOS 2021 – XI Latin and American Algorithms, Graphs and Optimization Symposium eventos.ufabc.edu.br/lagos2021/

