Estudando Emparelhamento e Combatendo Incêndios: Em Busca de Novos Limites em Teoria dos Grafos

remembering Vitor Costa (24/05/1981 - 31/12/2020)

Dieter Rautenbach
Universität Ulm

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Matchings in graphs of odd regularity and girth

Vitor Costa [a], Simone Dantas [a], Dieter Rautenbach [b,*]

[a] *Instituto de Matemática e Estatística, Universidade Federal Fluminense, Niterói, RJ, Brazil*

[b] *Institute of Optimization and Operations Research, University of Ulm, Ulm, Germany*

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Dedicated to Leonardo Gonzaga

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A B S T R A C T

We establish lower bounds on the matching number of graphs of given odd regularity d and odd girth g, which are sharp for many values of d and g. For $d = g = 5$, we characterize all extremal graphs.

Theorem 1. *Let r be a positive integer, and let g be an odd integer with $g \geq 5$. If G is a connected $(2r+1)$-regular graph of order n and odd girth at least g, then*

$$\alpha'(G) \geq \frac{1}{2}\left(n - \frac{o_{\mathrm{opt}}(2r+1,g)}{2r+1}\right), \tag{2}$$

where $o_{\mathrm{opt}}(2r+1,g)$ is the optimum value of the following integer linear program:

$$\begin{aligned} &\max \sum_{t\in T}(2r+1-t)o_t \\ &\textit{such that } \sum_{t\in T}\left((2r-1)n(2r+1,t,g)-t+2\right)o_t \leq (2r-1)n+2 \\ &o_t \in \mathbb{N}_0 \quad \textit{for } t \in T, \end{aligned} \tag{3}$$

where $T = \{t \in \mathbb{N} : 1 \leq t \leq 2r-1 \text{ and } t \text{ is odd}\}$.

Corollary 3. *Let r be a positive integer, and let g be an odd integer with $g \geq 5$. If G is a connected $(2r+1)$-regular graph of order n and odd girth at least g, then*

$$\alpha'(G) \geq \frac{1}{2}\left(n - \frac{4r((2r-1)n+2)}{(2r+1)\left((2r-1)((2r+1)g-1)+2\right)}\right). \tag{15}$$

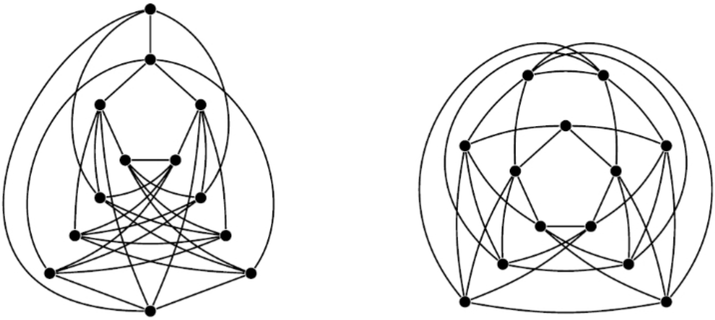

Fig. 1. The two (5, 1, 5)-extremal graphs.

Corollary 4. *If G is a connected 5-regular graph of order n and odd girth at least 5, then*

$$\alpha'(G) \geq \frac{47n-2}{100}, \tag{16}$$

with equality if and only if G arises from a tree T, where

- *$V(T)$ is the union of three independent sets X, R, and S,*
- *there are no edges between R and S,*
- *every vertex in $X \cup S$ has degree 5 in T, and*
- *every vertex in R has degree 1 in T,*

by adding $|R|$ disjoint factor-critical and (5, 1, 5)-extremal graphs G_u with $u \in R$ and identifying each vertex u in R with the unique vertex of degree 4 in G_u.

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More fires and more fighters

Vitor Costa [a], Simone Dantas [a], Mitre C. Dourado [b], Lucia Penso [c], Dieter Rautenbach [c,*]

[a] *Instituto de Matemática e Estatística, Universidade Federal Fluminense, Niterói, RJ, Brazil*

[b] *Instituto de Matemática, Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brazil*

[c] *Institute of Optimization and Operations Research, University of Ulm, Ulm, Germany*

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ABSTRACT

Hartnell's firefighter game models the containment of the spreading of an undesired property within a network. It is a one-player game played in rounds on a graph G in which a fire breaks out at f vertices of G. In each round the fire spreads to neighboring vertices unless the player defends these. The power of the player is limited in the sense that he can defend at most d additional vertices of G in each round. His objective is to save as many vertices as possible from burning. Most research on this game concerned the case $f = d = 1$, which already leads to hard problems even restricted to trees.

We study the game for larger values of f and d. We present useful properties of optimal strategies for the game on trees, efficient approximation algorithms, and bounds on the so-called surviving rate.

$$
\begin{aligned}
\text{maximize} \quad & \sum_{v \in L} \sum_{u : u \preceq v} x_u \\
\text{subject to} \quad & \sum_{u \in L_i} x_u \leq d_i \quad \text{for every } i \text{ with } 1 \leq i \leq n \\
& \sum_{u : u \preceq v} x_u \leq 1 \quad \text{for every } v \text{ in } L \\
& x_u \in \{0, 1\} \quad \text{for every } u \text{ in } L.
\end{aligned}
$$

Theorem 3.2. *Let f and d be two positive integers.*

There is a polynomial $(1 - 1/e)$-approximation algorithm for Max f-d-Firefighter *restricted to instances (T, F) where T is a tree.*

Theorem 4.1. *Let f and Δ be two positive integers.*

If T is a tree of maximum degree at most Δ, then $\rho(T, f; 1) \geq 1 - O\left(\frac{\log n(T)}{n(T)}\right)$ where the constants implicit in the $O(\cdot)$-notation only depend on f and Δ.

Theorem 5.3. *Let d be a positive integer, and let $0 \leq w_{\min} \leq w_{\max}$.*

There is a polynomial approximation algorithm with additive approximation guarantee $w_{\max} - w_{\min}$ for Max Weighted 1-d-Firefighter *restricted to instance $((G, w), \{r\})$ where G is a graph of maximum degree at most $d+2$, $w : V(G) \to [w_{\min}, w_{\max}]$, and r is a vertex of G of degree at most $d + 1$.*

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Asymptotic surviving rate of trees with multiple fire sources

Vitor Costa [a], Simone Dantas [a], Dieter Rautenbach [b,*]

[a] Instituto de Matemática e Estatística, Universidade Federal Fluminense, Niterói, RJ, Brazil

[b] Institute of Optimization and Operations Research, Ulm University, Ulm, Germany

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ABSTRACT

For Hartnell's firefighter game with f vertices initially on fire and at most d defended vertices per round, the surviving rate $\rho(G, f, d)$ of a graph G is the average proportion of its vertices that can be saved in the game on G, where the average is taken over all sets of f fire sources. Cai et al. (2010) showed that $\rho(T, 1, 1) = 1 - O\left(\frac{\log n}{n}\right) = 1 - o(1)$ for every tree T of order n.

We study the maximum value $c(f, d)$ such that $\rho(T, f, d) \geq c(f, d) - o(1)$ for every tree T of order n, where the $o(1)$ term tends to 0 as n tends to infinity. In this notation, the result of Cai et al. states $c(1, 1) = 1$. Our main results are that $c(f, 1) \geq \left(\frac{1}{3}\right)^f$ and that $\frac{4}{9} \leq c(2, 1) \leq \frac{3}{4}$.

The firefighter game is even hard for $f = d = 1$ and trees of maximum degree 3 [6] or cubic graphs [12]. Approximation algorithms for trees were considered in [10,2–4,11]. An interesting notion in this context is the *surviving rate of a graph G* [7] defined as

$$\rho(G, f, d) = \frac{1}{\binom{n(G)}{f}} \sum_{F \in \binom{V(G)}{f}} \frac{s(G, F, d)}{n(G)},$$

$$c(f, d) = \liminf_{n \to \infty} \Big(\min \{ \rho(T, f, d) : T \text{ is a tree of order } n \} \Big),$$

Theorem 2.5. *$c(f, 1) \geq \left(\frac{1}{3}\right)^f$ for every positive integer f.*

Theorem 2.6. $c(2, 1) \geq \frac{4}{9}$.

Let

$$f\left(x, y, (z_i)_{i \in [k]}\right) = x^2 + y^2 + xy(x + y) + \sum_{i \in [k]} 2(x + y) z_i (1 - z_i) + \sum_{i \in [k]} z_i^2 (1 - z_i) + \sum_{\{i,j\} \in \binom{[k]}{2}} 2 z_i z_j (1 - z_i - z_j). \quad (1)$$

Since increasing x by $O\left(\frac{1}{\sqrt{n}}\right)$ increases the value of $f(x, y, (z_i)_{i \in [k]})$ by $O\left(\frac{1}{\sqrt{n}}\right)$, we obtain $c(2, 1) \geq f_{\min}$, where $f_{\min}$ is the optimum value of the following optimization problem

$$\begin{aligned} \min \quad & f\left(x, y, (z_i)_{i \in [k]}\right) \\ s.th. \quad & x + y + z_1 + \cdots + z_k = 1 \\ & z_1, \ldots, z_k \leq \frac{1}{3} \\ & x, y, z_1, \ldots, z_k \geq 0. \end{aligned}$$

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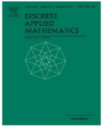

Slash and burn on graphs — Firefighting with general weights

CrossMark

Vitor Costa[a], Simone Dantas[a], Mitre C. Dourado[b], Lucia Penso[c], Dieter Rautenbach[c,*]

[a] *Instituto de Matemática e Estatística, Universidade Federal Fluminense, Niterói, RJ, Brazil*
[b] *Instituto de Matemática, Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brazil*
[c] *Institute of Optimization and Operations Research, University of Ulm, Ulm, Germany*

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ABSTRACT

In Hartnell's firefighter game a player tries to contain a fire breaking out at some vertex of a graph and spreading in rounds from burned vertices to their neighbors, by defending one vertex in each round, which will remain protected from the fire throughout the rest of the game. The objective of the player is to save as many vertices as possible from burning.

Here we study a generalization for weighted graphs, where the weights can be positive as well as negative. The objective of the player is to maximize the total weight of the saved vertices of positive weight minus the total weight of the burned vertices of negative weight, that is, the player should save vertices of positive weight and let vertices of negative weight burn. We prove that this maximization problem is already hard for binary trees and describe two greedy approximation algorithms for trees. Furthermore, we discuss a weighted version of the surviving rate.

Theorem 2.1. *For a given integer k and a given instance $((G, \omega), r)$ of* WEIGHTED FIREFIGHTER *where G is a binary tree with root r and $\omega : V(G) \to \{-1, 1\}$, it is NP-complete to decide whether there is a strategy σ with $f_{((G,\omega),r)}(\sigma) \geq k$.*

Input: An instance $((T, \omega), r)$ of WEIGHTED FIREFIGHTER where T is a tree.
Output: A strategy σ for $((T, \omega), r)$.

```
V ← V(T) \ {r};
i ← 1;
while V ≠ ∅ and max{pos(u) : u ∈ V} > 0 do
    choose g_i ∈ V such that pos(g_i) = max{pos(u) : u ∈ V};
    defend g_i in round i;
    remove from V all vertices in V(T(g_i));
    remove from V all vertices at distance i from r;
    i ← i + 1;
end
```

Theorem 3.1. *For an instance $((T, \omega), r)$ of* WEIGHTED FIREFIGHTER *where T is a tree, let*

- *σ_{opt} be an optimal strategy,*
- *$\sigma_{\text{pos-greedy}}$ be a strategy produced by* POS-GREEDY, *and*
- *$\sigma_{\emptyset}$ be the empty strategy not defending any vertex at all.*

We have

$$f(\sigma_{\text{opt}}) \leq 3 \max \left\{ f(\sigma_{\emptyset}), f(\sigma_{\text{pos-greedy}}) \right\},$$

that is, returning the better of the two strategies $\sigma_{\emptyset}$ and $\sigma_{\text{pos-greedy}}$ is a $\frac{1}{3}$-approximation algorithm.

Theorem 3.2. *For an instance $((T, \omega), r)$ of* WEIGHTED FIREFIGHTER *where T is a tree, let*

- *σ_{opt} be an optimal strategy and*
- *$\sigma_{\text{diff-greedy}}$ be a strategy produced by* DIFF-GREEDY.

We have

$$f(\sigma_{\text{opt}}) \leq 2 f(\sigma_{\text{diff-greedy}}),$$

that is, returning the strategy $\sigma_{\text{diff-greedy}}$ is a $\frac{1}{2}$-approximation algorithm.


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